

SOLUTION OF SHAPE IDENTIFICATION PROBLEM ON THERMOELASTIC SOLIDS WITH DESIRED DISTRIBUTION OF THERMAL DEFORMATION

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Abstract - This paper presents a numerical analysis method for solving the shape identification problems of thermoelastic fields. The square error integral between the actual thermal deformation distributions and the prescribed thermal deformation distributions on the prescribed sub-boundaries is used as the objective functional. The shape gradient of the shape identification problems was derived theoretically using the adjoint variable method, the Lagrange multiplier method and the formulae of the material derivative. Reshaping was accomplished using a traction method that was proposed as a solution to the domain optimization problems. A new numerical procedure for the shape identification was proposed. The validity of the proposed method was confirmed by the results of 2D numerical analysis.

1. INTRODUCTION

Boundary shape determinations, where the distributions of state functions, such as displacement or stress on linear elastic bodies, and temperature on heat-conduction fields, are specified with prescribed distributions for the purpose of improving the performance of machines and structures, are highly important issues in mechanical and structural design. In this study, we consider a shape identification problem which determines the boundary of the initial shape that after thermal deformation will be prescribed to the target shape on thermoelastic solids. These shape identifications are very important problems in the development of the machine tool with thermal deformation, and the shape design of the equipment for the purpose of improving machining accuracy by decreasing of the thermal deformation is one of the problems which connect with this study directly. Moreover, the establishment of such shape design technology is also desired in the development of the precise measurement equipment in which the thermal deformation influences error of measurement.

The thermoelastic problem is divided into two conventional problems, the weak coupled problem and the strong coupled problem. A problem in which the temperature distribution affects deformation, strain and stress is called the weak coupled problem. On the other hand, the types of problem to which the temperature distribution and deformation distribution affect each other is called the strong coupled problem. The design sensitivity analysis for thermoelastic problems was initiated by Meric[1] and Dems and Mroz[2]. Meric[1] analyzed the sensitivity on the weak coupled thermoelastic problem for linear, isotropic, and steady-state thermoelastic problems. Dems and Mroz[2] analyzed the sensitivity on the weak coupled thermoelastic problem for the problem taking into account the nonlinearity of the strain and temperature distributions. The sensitivity analysis of the strong coupled thermoelastic problem was carried out by Tortorelli *et al.*[3]. Hou *et al.*[4] and Bobaru and Mukherjee[5] proposed the numerical analyses methods for the shape determination problems of the linear weak coupled thermoelastic solids. Grindeanu *et al.*[6] analyzed the shape optimization problems of the weak coupled thermoelastic solids for durability, where Young's modulus and Poisson's ratio depend on the temperature. In their numerical analysis examples, however, only numerical analysis results for decreasing the number of design variable as much as possible have been reported.

On the other hand, the present authors have also focused on the solution of shape determination problems in fundamental single fields which do not take into consideration the coupling of the field, such as a linear elastic body, a heat conduction field, and a flow field. In previous papers, we presented numerical analysis methods for these shape identification problems of elastic bodies, heat-conduction fields and flow fields in which the square error integral between the actual distributions of state functions (displacement [7], temperature [8] [9], flow velocity [10] and pressure [11]) and the prescribed distributions on the sub-boundaries or in the sub-domains were used as the objective functional. Reshaping was accomplished by the traction method [12]-[14] which was proposed by one of the authors as a solution to shape optimization problems in which the boundary value problems were defined. The traction method is a method which consists of applying the gradient method of the distribution system which directly uses the shape sensitivity (shape gradient) of the domain variation which is theoretically derived from the optimization problem. In the traction method, the domain variations that minimize the objective functional are obtained as solutions of the pseudolinear elastic problems of continua defined on the design domains and loaded with pseudodistributed traction in proportion to the shape gradient on the design domains. The numerical solutions of both the shape gradient and the pseudolinear elastic problems, used for evaluation of the domain variation, can be obtained using the finite-element method or the boundary-element method. Therefore, the traction method can be applied to complex shape determination problems with a large number of design variables. Additionally, since this traction method is implemented by the use of the finite element method, it is exceptionally easy to perform and offers the advantage that it is not necessary to refine the mesh of the internal nodes of the domain.

In this study, we applied the traction method to the shape identification problem of the linear, steady-state weak coupled thermoelastic solids. We formulate a thermal deformation prescribed problem in which the square error integral between the actual thermal deformation distributions and the prescribed thermal deformation distributions on the prescribed sub-boundaries on the thermoelastic solids is used as the objective functional. The shape gradient of the shape identification problem was derived theoretically using the adjoint variable method, the Lagrange multiplier method and the formulae of the material derivative. Then, a numerical procedure using the finite element method for the shape identification problem was presented. The validity of the proposed method was confirmed by the results of a 2D numerical analysis.

2. DOMAIN VARIATION

Before formulating the shape identification problem, a method of representing domain variation using the speed method will be discussed briefly. A more detailed explanation is given in reference [15].

Assume that a domain $\Omega \subset R^n$, where $n = 2, 3$, R is the set of real numbers, and its boundary Γ , is variable. One approach to describing the domain variation is to use a one-parameter family of one-to-one mapping $T_s(X) : \Omega \ni X \mapsto x \in \Omega_s$, where s denotes the domain variation history.

When a domain functional J_Ω and a boundary functional J_Γ of a distributed function ψ are considered, their derivatives \dot{J}_Ω and \dot{J}_Γ with respect to s at $s = 0$ are given by the formulae of the material derivative:

$$J_\Omega = \int_\Omega \psi dx, \quad \dot{J}_\Omega = \int_\Omega \psi' dx + \int_\Gamma \psi \nu \cdot V dx, \quad (1)$$

$$J_\Gamma = \int_\Gamma \psi d\Gamma, \quad \dot{J}_\Gamma = \int_\Gamma \{\psi' + (\nabla_\nu \psi + \psi \kappa) \nu \cdot V\} d\Gamma, \quad (2)$$

where ν is an outward unit normal vector on the boundary, $\nabla_\nu(\cdot) \equiv \nabla(\cdot) \cdot \nu$ and κ denotes the $(n-1)$ times of the mean curvature. The shape derivative ψ' of the distributed function ψ indicates the derivatives under a spatially fixed condition. The derivative of $T_s(X)$ with respect to s given by

$$V(x) = \frac{\partial T_s}{\partial s} (T_s^{-1}(X)), \quad X \in \Omega, \quad x \in \Omega_s, \quad (3)$$

is called the velocity because of the analogy between s and time.

3. STATE EQUATIONS OF THERMOELASTIC SOLID

In this section, the variational formulations for the linear steady-state weakly coupled thermoelastic domain are represented.

3.1. Heat-conduction analysis

The variational form, or the weak form of the state equation of heat-conduction for the temperature ϕ using the adjoint temperature $\varphi \in \Phi$, is given as

$$a_\phi(\phi, \varphi) + b_\phi(\phi, \varphi) = l_\phi(\varphi) \quad \phi - \phi_0 \in \Phi, \quad \forall \varphi \in \Phi \quad (4)$$

where the terms $a_\phi(\phi, \varphi)$, $b_\phi(\phi, \varphi)$ and $l_\phi(\varphi)$ are defined by

$$\begin{aligned} a_\phi(\phi, \varphi) &= \int_{\Omega} k_{ij} \phi_{,i} \varphi_{,j} dx, & b_\phi(\phi, \varphi) &= \int_{\Gamma_h} h \phi \varphi d\Gamma \\ l_\phi(\varphi) &= \int_{\Omega} Q \varphi dx + \int_{\Gamma_h} h \phi_f \varphi d\Gamma + \int_{\Gamma_q} q \varphi d\Gamma. \end{aligned}$$

The set Φ of the temperature and adjoint temperature is given by

$$\Phi = \{\phi \in H^1(\Omega) \mid \phi|_{\Gamma_\phi} = 0, \Gamma_\phi \subset \Gamma\}. \quad (5)$$

The heat-conduction field is defined in a domain Ω with a boundary $\Gamma = \Gamma_\phi \cup \Gamma_h \cup \Gamma_q \cup \Gamma_i$. The temperature $\phi_0 \in H^{1/2}(\Gamma_\phi)$, and the heat flux $q \in H^{-1/2}(\Gamma_q)$ the heat source $Q \in H^{-1}(\Omega)$ are given as known functions. The boundary Γ_h is the heat transfer boundary obtained by the heat transfer coefficient $h \in L^\infty(\Gamma_h)$ and the ambient temperature $\phi_f \in L^\infty(\Gamma_h)$. $k = \{k_{ij}\}_{i,j=1}^n \in (L^\infty(\Omega))^{n \times n}$ is the thermal conductivity tensor, and the boundary Γ_i is the insulation boundary. The Einstein summation convention and partial differential notation $(\cdot)_{,i} = \partial(\cdot)/\partial x_i$ are used in the tensor notation throughout this paper. The symbols $L^\infty(\Omega)$ and $H^m(\Omega)^n$ denote the space of the bounded functions and the space of the square integrable functions until the m th derivatives, respectively.

3.2. Elastic analysis

The variational formulation for the state equation of the linear thermal elastic problem corresponding to the displacement $u = \{u_i\}_{i=1}^n \in U$ using the adjoint displacement $v \in U$, where thermal expansion based on the temperature distribution ϕ is considered, is given as

$$a_\varepsilon(\varepsilon(u) - \phi \alpha, \varepsilon(v)) = l_\varepsilon(v) \quad u - u_0 \in U, \quad \forall v \in U, \quad (6)$$

where the terms $a_\varepsilon(\varepsilon, \varepsilon)$, $\varepsilon_{ij}(u)$ and $l_\varepsilon(u)$ are defined by

$$\begin{aligned} a_\varepsilon(\varepsilon, \varepsilon) &= \int_{\Omega} C_{ijkl} \varepsilon_{kl} \varepsilon_{ij} dx, & \varepsilon_{ij}(u) &= \frac{1}{2}(u_{i,j} + u_{j,i}) \\ l_\varepsilon(u) &= \int_{\Omega} f_i u_i dx + \int_{\Gamma_P} P_i u_i d\Gamma. \end{aligned}$$

The set U of the displacement and adjoint displacement is given by

$$U = \{u \in (H^1(\Omega))^n \mid u|_{\Gamma_u} = 0, \Gamma_u \subset \Gamma\}. \quad (7)$$

The elastic field is defined in a domain Ω with a boundary $\Gamma = \Gamma_u \cup \Gamma_P$. The displacement $u_0 = \{u_{oi}\}_{i=1}^n \in (H^{1/2}(\Gamma_u))^n$, the surface force $P = \{P_i\}_{i=1}^n \in (H^{-1/2}(\Gamma_P))^n$ and the body force $f = \{f_i\}_{i=1}^n \in (H^{-1}(\Omega))^n$ are known functions, respectively. The symbols $C = \{C_{ijkl}\}_{i,j,k,l=1}^n \in (L^\infty(\Omega))^{n \times n}$ and $\alpha = \{\alpha_{ij}\}_{i,j=1}^n \in (L^\infty(\Omega))^{n \times n}$ are the material stiffness tensor and the thermal expansion coefficient tensor, respectively.

4. SQUARE ERROR INTEGRAL MINIMIZATION PROBLEM OF THERMAL DEFORMATION

4.1. Formulation of problem

We consider a prescribed thermal deformation problem in which the square error integral between the actual thermal displacement u and the prescribed thermal displacement u_D on the prescribed sub-boundaries $\Gamma_D \subset \Gamma$ of the thermoelastic domain Ω is used as the objective functional. This problem is formulated as

$$\text{Given } M \text{ and} \quad (8)$$

$$k, \phi_o, q, h, \phi_f, Q, C, u_o, \alpha, P, f : \text{ fixed in space} \quad (8)$$

$$\text{find } \Omega \quad (9)$$

$$\text{that minimize } E(u - u_D, u - u_D) \quad (10)$$

$$\text{subject to } a_\phi(\phi, \varphi) + b_\phi(\phi, \varphi) = l_\phi(\varphi) \quad \phi - \phi_0 \in \Phi, \quad \forall \varphi \in \Phi \quad (11)$$

$$a_\varepsilon(\varepsilon(u) - \phi\alpha, \varepsilon(v)) = l_\varepsilon(v) \quad u - u_0 \in U, \quad \forall v \in U \quad (12)$$

$$\int_{\Omega} dx \leq M, \quad (13)$$

where the square error integral $E(x, y)$ is defined by

$$E(x, y) = \int_{\Gamma_D} x_i y_i d\Gamma.$$

Equations (11) and (12) are the state equations of the thermoelastic problem, and eqn.(13) is volume constraint condition.

4.2. Shape gradient

Applying the concept of the Lagrange multiplier method and the adjoint variable method, this problem can be rendered as a stationary problem for the Lagrange functional $L(\phi, u, \varphi, v, \Lambda)$:

$$\begin{aligned} L(\phi, u, \varphi, v, \Lambda) = & \\ & E(u - u_D, u - u_D) - a_\phi(\phi, \varphi) - b_\phi(\phi, \varphi) + l_\phi(\varphi) \\ & - a_\varepsilon(\varepsilon(u) - \phi\alpha, \varepsilon(v)) + l_\varepsilon(v) + \Lambda \left(\int_{\Omega} dx - M \right), \end{aligned} \quad (14)$$

where the nonnegative real number Λ is the Lagrange multiplier with respect to the volume constraint. For simplicity, when assuming that the heat flux sub-boundary Γ_q , the heat transfer sub-boundary Γ_h , the surface force sub-boundary Γ_P and the prescribed displacement sub-boundary Γ_D are invariable under reshaping, the derivative \dot{L} with respect to s is derived using eqns (1) and (2):

$$\begin{aligned} \dot{L} = & 2E(u - u_D, u') \\ & - a_\phi(\phi, \varphi') - b_\phi(\phi, \varphi') + l_\phi(\varphi') \\ & - a_\phi(\phi', \varphi) - b_\phi(\phi', \varphi) \\ & - a_\varepsilon(\varepsilon(u) - \phi\alpha, \varepsilon(v')) + l_\varepsilon(v') \\ & - a_\varepsilon(\varepsilon(u'), \varepsilon(v)) + a_\varepsilon(\phi'\alpha, \varepsilon(v)) \\ & + \dot{\Lambda} \left(\int_{\Omega} dx - M \right) \\ & + \langle G\nu, V \rangle, \end{aligned} \quad (15)$$

where $(\dot{\cdot})$ is the material derivative and $(\cdot)'$ is the shape derivative for the domain variation of the distributed function under a spatially fixed condition. The linear form $\langle G\nu, V \rangle$ with respect to the velocity function V is given by

$$\langle G\nu, V \rangle = \int_{\Gamma} G\nu_i V_i d\Gamma \quad (16)$$

$$G = G_0 + G_1 \Lambda, \quad (17)$$

where

$$G_0 = -C_{ijkl}(\varepsilon_{kl}(u) - \alpha_{kl}\phi)\varepsilon_{ij}(v) - k_{ij}\phi_{,i}\varphi_{,j} + Q\varphi + f_i v_i \quad (18)$$

$$G_1 = 1. \quad (19)$$

Considering the stationary conditions for all $\varphi' \in \Phi$, $v' \in U$, $u' \in U$, $\phi' \in \Phi$ and Λ from eqn.(15), the Kuhn-Tucker conditions with respect to ϕ, u, v, φ are obtained as

$$a_\phi(\phi, \varphi') + b_\phi(\phi, \varphi') = l_\phi(\varphi') \quad \forall \varphi' \in \Phi \quad (20)$$

$$a_\varepsilon(\varepsilon(u) - \phi\alpha, \varepsilon(v')) = l_\varepsilon(v') \quad \forall v' \in U \quad (21)$$

$$a_\varepsilon(\varepsilon(u'), \varepsilon(v)) = 2E(u - u_D, u') \quad \forall u' \in U \quad (22)$$

$$a_\phi(\phi', \varphi) + b_\phi(\phi', \varphi) = a_\varepsilon(\phi'\alpha, \varepsilon(v)) \quad \forall \phi' \in \Phi \quad (23)$$

$$\Lambda \geq 0, \quad \int_\Omega dx \leq M, \quad \Lambda \left(\int_\Omega dx - M \right) = 0, \quad (24)$$

that indicate variational formulation for the original state equations for the temperature ϕ and displacement u , the variational forms of the adjoint equations for adjoint temperature φ and adjoint displacement v . The adjoint displacement v in eqn.(22) can be evaluated as the displacement by a loading of a pseudo force $2(u - u_D)$ on the prescribed displacement sub-boundary Γ_D in the linear elastic domain Ω . The right side term $a_\varepsilon(\phi'\alpha, \varepsilon(v))$ of eqn.(23) is rewritten as

$$a_\varepsilon(\phi'\alpha, \varepsilon(v)) = \int_\Omega S\phi' dx = \int_\Omega C_{ijkl}\varepsilon_{kl}(v)\alpha_{ij}\phi' dx. \quad (25)$$

Therefore, the adjoint temperature φ in Eq.(23) can be evaluated as a temperature by the generation of a pseudo heat source $S = C_{ijkl}\varepsilon_{kl}(v)\alpha_{ij}$ in the heat conduction domain Ω .

Under the condition satisfying eqns (20)-(24), the derivative of the Lagrange functional agrees with that of the objective functional and the linear form $\langle G\nu, V \rangle$ with respect to the velocity function V :

$$\dot{L}|_{\phi, u, \varphi, v, \Lambda} = \langle G\nu, V \rangle. \quad (26)$$

The coefficient vector $G\nu$ in eqn.(16) has the meaning of a sensitivity relation to the domain variation and is called the shape gradient. The scalar function G is called the shape gradient density.

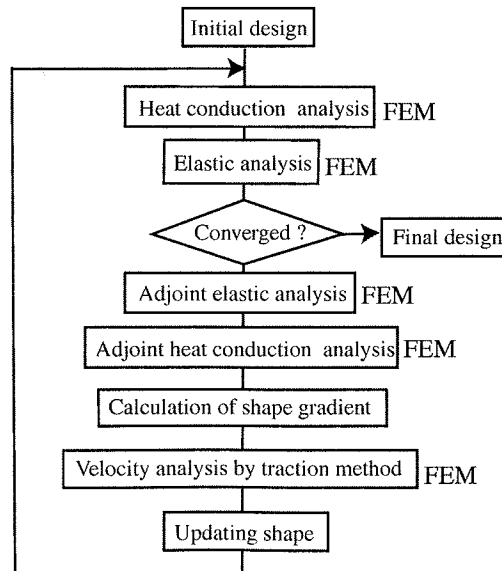


Figure 1: Flow chart of shape identification.

5. NUMERICAL SOLUTION TECHNIQUE

5.1. Traction method

When the shape gradient is obtained, the traction method [12]-[14] can be applied to optimize the geometrical domain shape. The traction method has been proposed as a procedure for solving the velocity $V \in D$ by

$$a_\varepsilon(\varepsilon(V), \varepsilon(y)) = - \langle G\nu, y \rangle \quad (27)$$

and

$$D = \{V \in (H^1(\Omega))^n \mid \text{constraints of domain variation} \}. \quad (28)$$

Equation (27) indicates that the velocity V decreasing the objective functional is obtained as a displacement of a pseudoelastic body defined in Ω by the loading of a pseudo-external force in proportion to $-G\nu$, under constraints on the displacement of invariable boundaries. Robustness of the traction method against oscillating phenomena, which often occur in the moving boundary nodes of finite element models in proportion to the negative value of the shape gradient, has been confirmed theoretically [13].

5.2. Numerical procedure

A flow chart of the shape identification system for thermoelastic solids is shown in Figure 1. The finite element method was employed in every analysis. Shape identification analysis was performed by executing these elements sequentially and repeatedly.

The shape gradient was evaluated using the two thermal elastic field analyses which analyze distributions of the temperature ϕ and the displacement u for the original state eqns (20) and (21), and the distributions of adjoint displacement v and adjoint temperature φ for the adjoint eqns (22) and (23). The domain variation V in eqn.(27) was analyzed using the finite element method. The Lagrange multiplier Λ , determined so as to satisfy the volume constraint, can be regarded as a uniform surface force in the external force $-G\nu$. Therefore, it should be possible to satisfy the conditions of eqn.(24) by controlling the magnitude of this uniform surface force Λ .

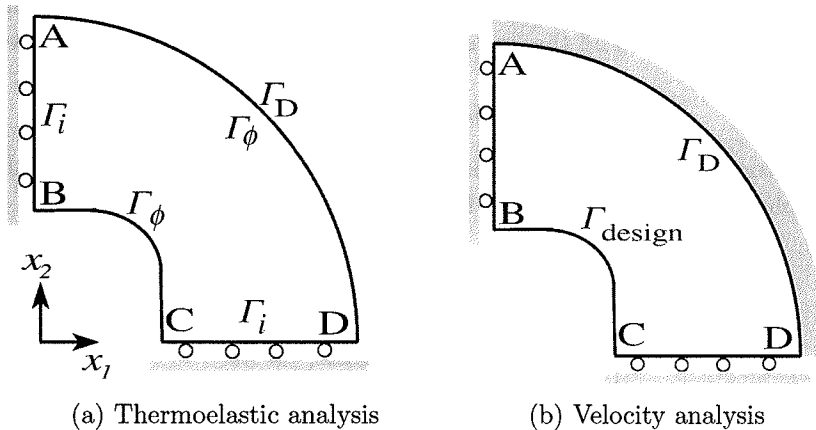


Figure 2: 2D Cylindrical continuum problem (quarter domain).

6. NUMERICAL RESULTS

We present the results of two numerical analyses for 2D shape identification problems using the traction method and shape gradient derived as described in previous sections.

6.1. Cylindrical continuum

We analyzed the shape identification problem of the cylindrical 2D continuum problem as shown in Figure 2. The quarter-symmetric domain was used in this problem. The outer surface boundary A-B was assumed as the prescribed displacement boundary Γ_D . The design boundary Γ_{design} is the inner boundary B-C. The central shape shown in Figure 3 was chosen to be the target shape, and the displacement distribution on the outer surface boundary Γ_D was assumed to be the prescribed displacement distribution u_D .

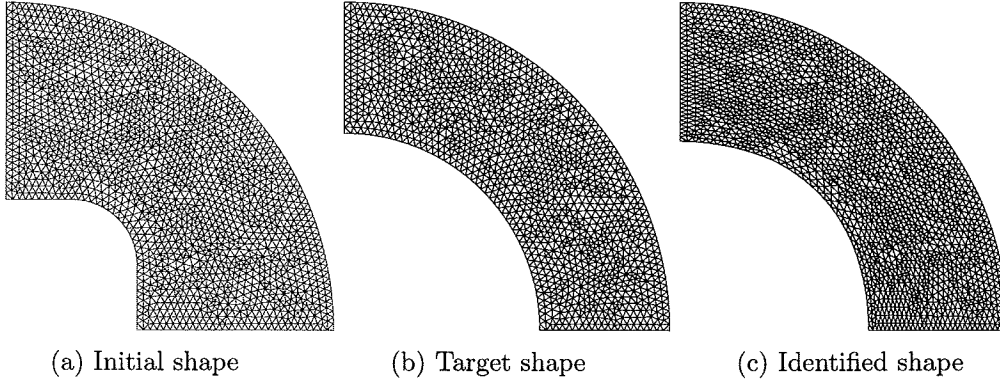


Figure 3: Shapes with finite element meshes in cylindrical 2D continuum problem.

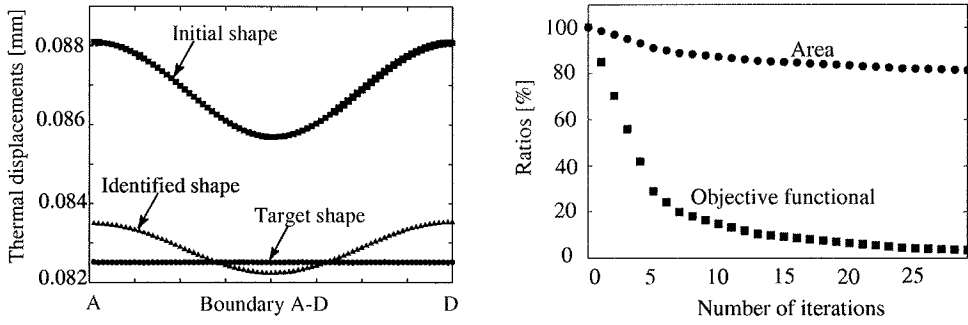


Figure 4: Thermal displacement distribution $\sqrt{u_1^2 + u_2^2}$ on boundary A-D (left side) and iterative history of objective functional (right side) in 2D cylindrical continuum problem.

For the heat conduction analysis, the outer and inner surface boundaries were assumed as the specified temperature boundary Γ_ϕ , where $\phi = 200^\circ\text{C}$ on the boundary A-D and $\phi = 50^\circ\text{C}$ on the boundary B-C. The boundaries A-B and C-D are the insulation boundary Γ_i . For the elastic analysis, the boundaries A-B and C-D were assumed as the slide boundary. The boundary A-D was fixed, and the boundaries A-B and C-D were assumed as slide boundary in the velocity analysis. For simplicity, the case we considered had the following conditions: length of boundary A-B= 20mm, Young's modulus 206GPa, Poisson's ratio 0.3, heat-conductivity coefficient $k_{ij} = k\delta_{ij} = 50\text{W/m}$, and thermal expansion coefficient $\alpha_{ij} = \alpha\delta_{ij} = 1.2 \times 10^{-5}\text{m/m}\cdot^\circ\text{C}$. This problem was analyzed using a plane strain condition.

Numerical results are shown in Figures 3 and 4. Figure 3 shows a comparison for the initial shape, target shape and identified shape with the finite element meshes. The left-hand shape shown in Figure 3 is chosen as an initial shape, and the identified shape analyzed, as the size of its area would consist of 80% or less of the initial shape area, is shown on the right-hand side of Figure 3. The left side of Figure 4 shows a comparison of the displacement distribution, which is $\sqrt{u_1^2 + u_2^2}$ on the outer boundary A-D for the target shape, initial shape and identified shape. The right side of Figure 4 shows the iterative history ratio of the objective functional $E(u - u_D, u - u_D)$ normalized with the initial value.

On the basis of these results, it was confirmed that the value for the objective functional approached zero, and the identified shape analyzed by the proposed method exhibits a good agreement with the target shape shown in Figure 3.

6.2. Plate with two holes

The proposed method was applied to the shape identification problem of a plate with two holes, as shown in Figure 5. The upper surface boundary A-D was assumed as the prescribed displacement boundary Γ_D . The design boundaries Γ_{design} are the left and right sides and also the boundaries of the two holes. The purpose of this analysis is to determine the shape in which the thermal deformation distribution in the x_2 direction at surface A-B become as uniform as possible. For simplicity, the case we considered had the following conditions: specified temperature $\phi = 100^\circ\text{C}$ on boundary A-D and 0°C on boundary B-C; length of the boundary A-B, = 200mm; and other material properties, the same as those in the previous problem. We analyzed two cases: the surface forces are not imposed (Case 1: $P = 0$), and they are imposed (Case 2: $P = 500\text{kN}$) on the boundary B-C.

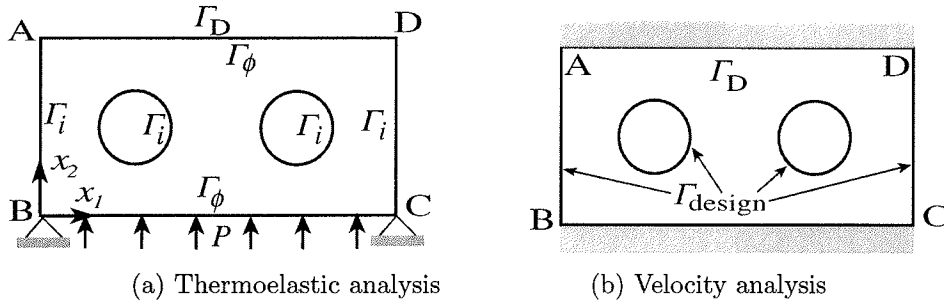


Figure 5: 2D plate problem with two holes.

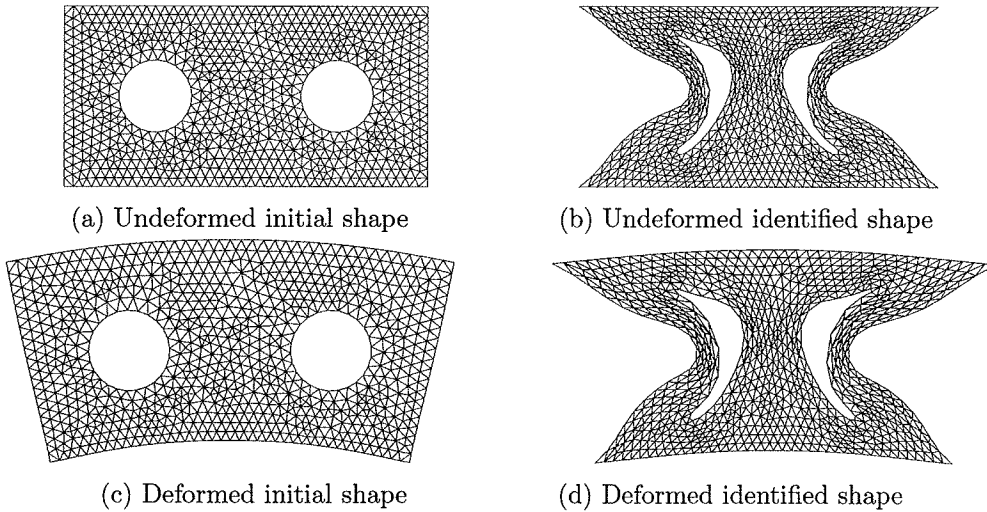


Figure 6: Shapes with finite element meshes in 2D plate problem with two holes (Case 1: $P = 0$).

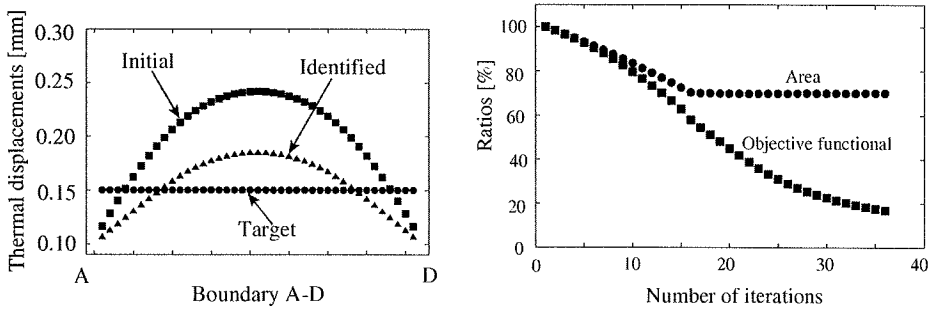


Figure 7: Thermal displacement distribution u_2 on boundary A-D (left side) and iterative history of objective functional (right side) in 2D plate problem with two holes (Case 1: $P = 0$).

Numerical results are shown in Figures 6-9 in the same manner as the results for the previous problem. The area consists of 70% or less of the initial shape area for the Case 1 problem. In the Case 2 problem, the area consists of the same initial shape area or less. The finite elements of the identified shapes in Figure 6 are severely distorted. As we have described in the introduction, the traction method has the advantage of not requiring mesh refinement for the internal nodes of the domain in the conventional-shape optimization analyses. Therefore, in these numerical analyses, we did not use re-mesh processing. From the right-hand sides of Figures 7 and 9, it was confirmed that the objective functional converged at the minimum value. The left-hand sides of Figures 7 and 9 show the thermal displacement u_2 in the x_2 direction on the upper surface boundary A-D between the initial and identified states. From both these results, the difference between the maximum displacement and the minimum displacement on the surface boundary became small, and an improvement of about 40% was obtained for the initial state.

We further analyzed another problem Case 3, in which the prescribed thermal deformation distribution is not an uniform; for this case, the identified shape obtained under the conditions of Case 2 was used as a target distribution u_D on the surface boundary A-D. The initial shape in Case 3 is same as that in

Case 2. The numerical results of the analysis are shown in Figures 10 and 11. From these results, it was confirmed that the thermal displacement distribution for the identified shape exhibits a good agreement with the target distribution u_D , and the objective functional approached zero, if the prescribed thermal displacement distribution which is not uniform would be chosen as u_D , similar to that in Case 3.

According to the numerical results obtained for the above basic problems, the validity of the present method was confirmed.

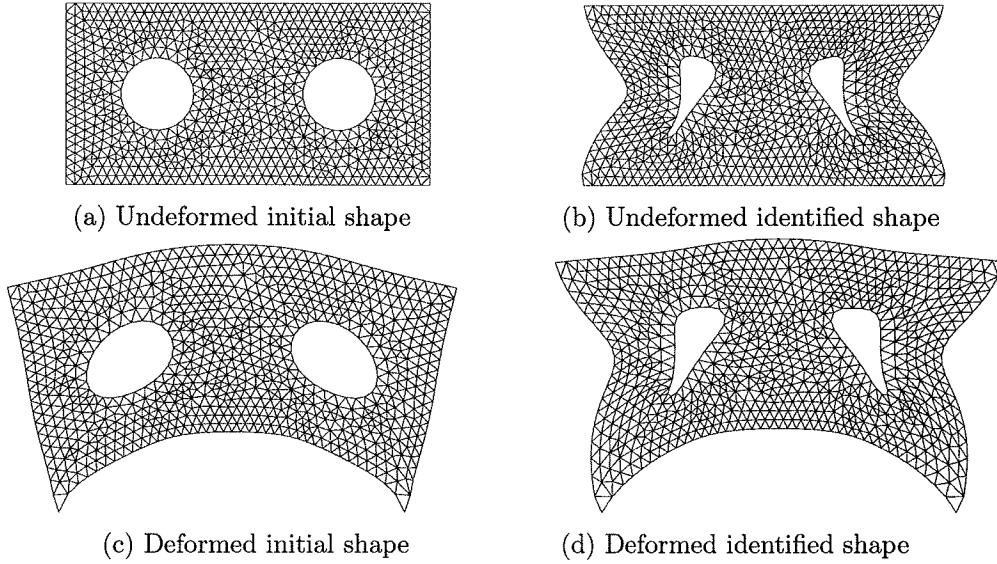


Figure 8: Shapes with finite element meshes in 2D plate problem with two holes (Case 2: $P = 500kN$).

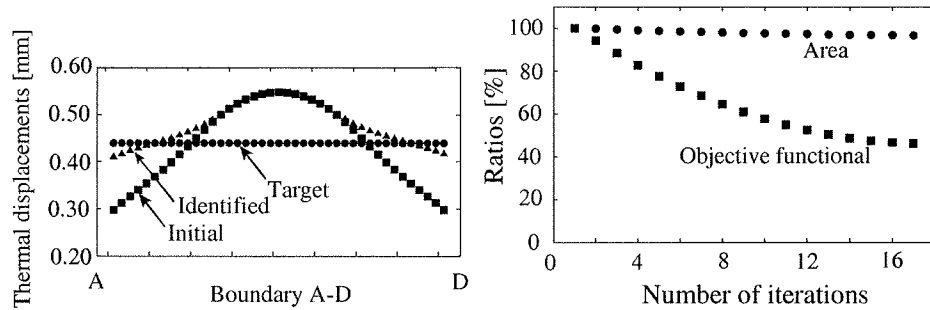


Figure 9: Thermal displacement distribution u_2 on boundary A-D (left side) and iterative history of objective functional (right side) in 2D plate problem with two holes (Case 2: $P = 500kN$).

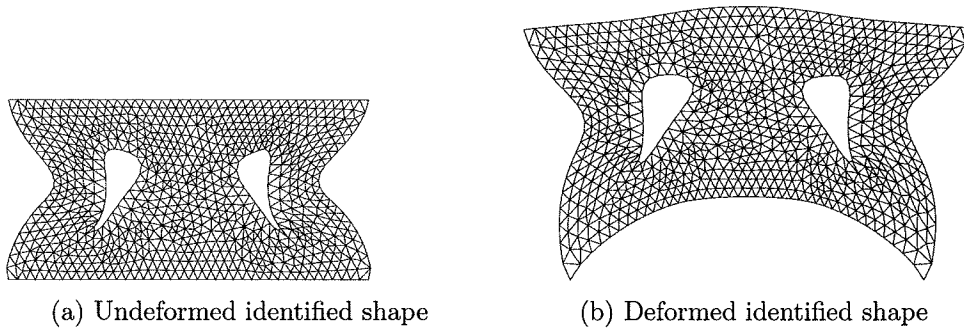


Figure 10: Shapes with finite element meshes in 2D plate problem with two holes (Case 3: $P = 500kN$).

7. CONCLUSIONS

In this study, we formulated a shape identification problem in which the square error integral between the actual thermal deformation distributions and the prescribed thermal deformation distributions on the prescribed sub-boundaries on the thermoelastic solids is used as objective functional. The shape gradient of the shape identification problem was derived theoretically. A numerical procedure using the

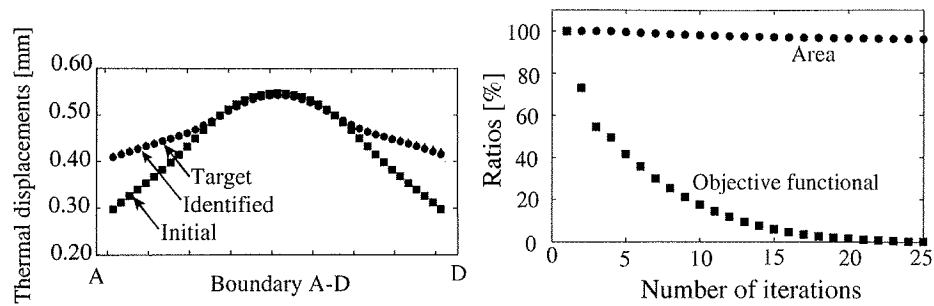


Figure 11: Thermal displacement distribution u_2 on boundary A-D (left side) and iterative history of objective functional (right side) in 2D plate problem with two holes (Case 3: $P = 500kN$)

derived shape gradient and the traction method was proposed. The validity of the proposed method was confirmed on the basis of the results of a 2D numerical analysis.

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